Numerical Simulation of Ethanol–Water–NaCl Droplet Evaporation

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A quantitative description of droplet evaporation is important to aerosol research for nanofabrication, spray drying, fuel combustion, pollution control, and respiratory medical treatments. Evaporation is a moving-boundary problem with coupled mass and heat transport. An explicit finite-difference methodology and computer code have been developed for simulation of an evolving droplet, property data for size, and profiles for various compositions and temperature. The code accurately predicts the evaporation of pure water and pure ethanol droplets. To understand aerosol-assisted evaporation-induced self-assembly and the formation mechanism for single-crystal NaCl core/hexagonally ordered mesoporous silica shell particles, evaporation of ethanol–water–NaCl droplets in N2 has been investigated by numerical simulation. The extended universal quasichemical (UNIQUAC) model with a Debye–Hückel term is used to describe the vapor–liquid phase equilibrium. For 1–2-µm-radius droplets with a number density of 10^7/cm^3, it takes only tens of milliseconds to reach phase equilibrium after adiabatic or isothermal evaporation at 25 °C in the drying zone. The droplets entering a heating zone can be simply treated like a single-stage flash evaporation at 25 °C. For a 1-µm-radius droplet, after 0.18 ms of evaporation at 100 °C in N2, the NaCl saturation ratio reaches levels as high as 1.3, first at the droplet center, where the initial NaCl nucleation and crystallization happens as a result of relatively quick evaporation and a steep gradient in the concentration of ethanol, an antisolvent for NaCl. NaCl crystallization “consumes” NaCl molecules near the droplet center and quenches the formation of new stable NaCl nuclei, favoring the formation of only one single-crystal NaCl core per droplet. The code provides guidance for the custom engineering of aerosol nanoparticle architectures.

Introduction

Aerosols have been the subject of growing importance to a broad spectrum of applications including spray drying, combustion, pollution control, microelectronics, occupational hygiene, and biomedical technology. Material synthesis using spray and aerosol processes has been demonstrated for a wide variety of materials, including drug delivery platforms, ceramic and metallic powders, and carbonaceous materials, etc. Spray and aerosol synthesis of particles involves solvent evaporation from precursor droplets, usually followed or accompanied by precursor reactions to produce the desired product. It has also been demonstrated that solvent evaporation can be exploited to drive the self-assembly of amphiphilic molecules within inorganic precursor solutions (evaporation-induced self-assembly or EISA), leading to oxide particles possessing templated mesoporosity with remarkable control over the pore size and mesostructural order. The use of aerosol-assisted EISA has also been shown to be a powerful strategy for producing particles with hierarchical porosity and multilayer composites, with a potential value as controlled release vehicles.

The concentration gradients that develop within droplets during solvent evaporation play a critical role in the morphology of particles produced by spray and aerosol methods. The production of hollow particles in spray and aerosol processing is very well-known, and the basic evaporation theory explaining the synthesis of hollow particles from simple precursor solutions has been well documented. For a single precursor solution, solvent evaporation from the droplet surface leads to a higher concentration of the precursor compound at the droplet surface, with a decreasing concentration going toward the center of the droplet. In this traditional theory, the rate of solvent evaporation affects the steepness of the concentration gradients inside the droplet; solid precipitation initiates when the critical supersaturation is reached for the precursor compound. On the basis of the diffusion equation modified by van der Lijn, Jayanthi et al. investigated the evaporation stage of solution aerosol thermolysis and simulated concentration profiles inside the droplet and proposed two criteria for solid particle formation. In a simple precursor solution, solid formation is thus predicted to initiate at the droplet surface where the precursor concentration is highest, and depending on the relative concentrations of supersaturation and saturation (solubility), hollow or nonhollow solid particle formation may occur.

The physical picture of this process becomes considerably more complex in droplets that contain multiple solvents and/or multiple precursors. This is apparent, for example, in the recent observation of single NaCl crystals formed within the interior of mesoporous silica, produced by EISA of aerosols, where the initial nucleation apparently occurred at or near the center of an evaporating droplet. Understanding the behavior of precursor systems such as the ones that contain multiple solvents and/or precursor compounds requires consideration of a rigorous thermodynamic model, coupled with multicomponent evaporation, diffusion, and energy transport models. So far, prediction of the droplet size evolution and droplet equilibrium temperature still relies on empirical correlations based on experimental data, and the correlations are available only for a limited number of solvents. We lack a detailed understanding of the temporal evolution even for pure water droplets in air. The complexity and coupled nature of such problems require numerical methods.
of solution. Many methods have been tried to solve the one-dimensional (1D) time-dependent moving-boundary problem. For example, Asaithambi\textsuperscript{20} proposed a variable time-step finite-element method. Caldwell et al.\textsuperscript{21} provided benchmarking for the use of nodal integral and finite-difference solution of the 1D Stefan problem. The partial differential equations for the problem with implicit boundary conditions were discretized using an implicit finite-difference scheme.\textsuperscript{22} These methods have been developed for specific problems and converge under special boundary and initial conditions.

In this paper, we have utilized a moving-boundary finite-difference method to study the evaporation of ethanol–water–NaCl droplets. We demonstrate how solvent-based solubility changes can influence the location of precipitation or solid formation, and hence the particle structure and morphology. These simulations are compared with our experimental observations for the same precursor system.

Theory and Model Development

For most evaporation phenomena, the Knudsen number, a dimensionless number defined as the ratio of the molecular mean-free path length to the droplet size, falls within the range $10^{-3}–10^{-2}$, and the evaporation system falls within the continuum regime, where the evaporation is controlled by diffusion and the coexisting phases can be assumed at equilibrium. This liquid–vapor phase equilibrium is expressed as

$$x_iP^s_i\gamma_iK_{R,i} = y_iP$$

where $P$, $P^s$, $\gamma$, $x_i$, and $y_i$ are respectively the gas-phase total pressure and saturated vapor pressure, activity coefficient, liquid mole fraction, and vapor-phase mole fraction for component $i$. The vapor phase will be assumed as an ideal gas ($q^v_i = 1$). $K_{R,i}$ is the Kelvin factor. For large droplets ($>1$ $\mu$m), the Kelvin effect can be neglected. The mass transport of an isotropic diffusion system is described by the equation of continuity for species $i$.

$$\frac{\partial C_i}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 D_i \frac{\partial C_i}{\partial r} \right)$$

Similarly, the heat transport can be described as

$$\frac{\rho C_p \partial T}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 k \frac{\partial T}{\partial r} \right)$$

where $t$ is the evaporation time, $C_i(r)$ is the local concentration of component $i$, $T$ is the absolute temperature, $C_p$ is the heat capacity at constant pressure, $k$ is the thermal conductivity, and $\rho$ is the mass density. Equations 2 and 3 are valid only for the evaporation without radial convection.

Approach

The above partial differential equations are difficult to solve simultaneously because all of the physical properties are interconnected and change with both time and location. To solve the moving-boundary problem, van der Lijn\textsuperscript{18} derived the following form for a binary system using a reference-component-centered coordinate system.

$$\frac{D}{Dt}(C_i/C_r) = \frac{1}{C_r r^2} \frac{\partial}{\partial r} \left[r^2 C_r D_i \frac{\partial (C_i/C_r)}{\partial r} \right]$$

For a multisolvent system containing nonvolatile solutes, this is too complicated to be practical. Moreover, eq 4 still contains a substantial derivative. The validity of eq 4 relies on the assumption that there is no flux for the reference component across the moving boundary. For pure or multicomponent solvent evaporation, no such reference component exists and this method does not apply. By mathematical manipulation, one can easily derive a valid eq 5 to replace eq 2 and convert the moving-boundary problem to a conventional fixed-boundary problem. In eq 5, $R$ is a dimensionless radius, $r/R_0$, and $R_0$ is the droplet size, which is an implicit function of time.
of the numerical treatment have been published previously,\textsuperscript{14,23} the vapor phase of size \( N \) into the initial and boundary conditions for component \( R \) are divided into a large number of concentric control elements and details of the numerical treatment have been published previously,\textsuperscript{14,23} but highlights are reviewed here for completeness.

As shown in Figure 1, both the droplet and surrounding gas phase are divided into a large number of concentric control shells. For fast numerical simulation, the liquid phase is divided into \( N_l \) concentric uniform or nonuniform control elements and the vapor phase of size \( R_v \) (\( R_v \rightarrow \infty \) for infinite air) is separated into \( N_v \) concentric control elements.

Equations 2 and 3 are solved by a finite-difference method. Considering a uniform initial concentration distribution, the initial and boundary conditions for component \( i \) are given as follows:

\[
C(r, 0) = C_{i,0} \quad \text{for} \quad 0 \leq r \leq R_{L,0} 
\]

\[
C(r, 0) = C_{i,0,V} \quad \text{for} \quad R_{L,0} < r \leq R_{V,0} 
\]

\[
\left( \frac{\partial C}{\partial r} \right)_{r=0} = 0
\]

\[
\left( \frac{\partial C}{\partial r} \right)_{R_v} = 0 \quad \text{or} \quad C_i = C_{i,\infty} \quad \text{at} \quad R = \infty
\]

Although this equation is quite suitable for numerical simulation and no substantial derivative is present, it is problematic that one assumes that the multicomponent droplet keeps the same shrinkage ratio everywhere and that each element keeps a fixed dimensionless radius during evaporation. For the evaporation of droplets of initial radius \( R_{L,0} \) in a still \( N_2 \) volume of equivalent radius \( R_{V,0} \) determined by the aerosol droplet number density, the initial and boundary conditions for component \( i \) and details of the numerical treatment have been published previously,\textsuperscript{14,23} but highlights are reviewed here for completeness.

\[
R_{L} \left( \frac{\partial C_i}{\partial t} \right)_{R} = \frac{1}{R^2} \frac{\partial}{\partial R} \left( D_i R^2 \frac{\partial C_i}{\partial R} \right) \quad (5)
\]

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\]

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\]

\[
\left( \frac{\partial C_i}{\partial r} \right)_{R_v} = 0 \quad \text{or} \quad C_i = C_{i,\infty} \quad \text{at} \quad R = \infty
\]

\[
T(r, 0) = T_{0,V} \quad \text{for} \quad R_{L,0} < r \leq R_{V,0} 
\]

\[
\left( \frac{\partial T}{\partial r} \right)_{r=0} = 0
\]

\[
\left( \frac{\partial T}{\partial r} \right)_{R_v} = 0 \quad \text{or} \quad T \big|_{r=\infty} = T_{\infty}
\]

The mass balance for component \( i \) in the liquid layer (NL) at the interface during the time interval between \( t \) and \( t + \Delta t \) can be written as

\[
R_{NL-1} \left( \frac{\partial C_{i,NL}}{\partial t} \right)_{NL} - R_{NL,-1} \left( \frac{\partial C_{i,NL}}{\partial t} \right)_{NL} = 2R_{NL} D_{i,NL} \left( C_{i,NL+1} - C_{i,NL-1} \right) - R_{NL} \left( C_{i,NL} - C_{i,NL,eq} \right)
\]

\[
R_{NL-1} \left( \frac{\partial C_{i,NL}}{\partial t} \right)_{NL} - R_{NL,-1} \left( \frac{\partial C_{i,NL}}{\partial t} \right)_{NL} = 2R_{NL} D_{i,NL} \left( C_{i,NL+1} - C_{i,NL-1} \right) - R_{NL} \left( C_{i,NL} - C_{i,NL,eq} \right)
\]

For the first vapor element

\[
R_{NL} \left( \frac{\partial C_{i,NL+1}}{\partial t} \right)_{NL} - R_{NL+1} \left( \frac{\partial C_{i,NL+1}}{\partial t} \right)_{NL} = 2R_{NL} D_{i,NL} \left( C_{i,NL+2} - C_{i,NL+1} \right) - R_{NL+1} \left( C_{i,NL+2} - C_{i,NL+1} \right)
\]

\[
R_{NL} \left( \frac{\partial C_{i,NL}}{\partial t} \right)_{NL} - R_{NL+1} \left( \frac{\partial C_{i,NL}}{\partial t} \right)_{NL} = 2R_{NL} D_{i,NL} \left( C_{i,NL} - C_{i,NL+1} \right) - R_{NL+1} \left( C_{i,NL} - C_{i,NL+1} \right)
\]

Similarly, the energy balance for the liquid layer (NL) can be written as the following equation:

\[
R_{NL} \left( \frac{\partial T_{NL}}{\partial t} \right)_{NL} - R_{NL+1} \left( \frac{\partial T_{NL}}{\partial t} \right)_{NL} = 2R_{NL} H_{i,NL} \left( T_{NL+1} - T_{NL} \right) - R_{NL+1} \left( T_{NL+1} - T_{NL} \right)
\]

\[
R_{NL} \left( \frac{\partial T_{NL}}{\partial t} \right)_{NL} - R_{NL+1} \left( \frac{\partial T_{NL}}{\partial t} \right)_{NL} = 2R_{NL} H_{i,NL} \left( T_{NL+1} - T_{NL} \right) - R_{NL+1} \left( T_{NL+1} - T_{NL} \right)
\]

\[
R_{NL} \left( \frac{\partial T_{NL}}{\partial t} \right)_{NL} - R_{NL+1} \left( \frac{\partial T_{NL}}{\partial t} \right)_{NL} = 2R_{NL} H_{i,NL} \left( T_{NL+1} - T_{NL} \right) - R_{NL+1} \left( T_{NL+1} - T_{NL} \right)
\]

\[
R_{NL} \left( \frac{\partial T_{NL}}{\partial t} \right)_{NL} - R_{NL+1} \left( \frac{\partial T_{NL}}{\partial t} \right)_{NL} = 2R_{NL} H_{i,NL} \left( T_{NL+1} - T_{NL} \right) - R_{NL+1} \left( T_{NL+1} - T_{NL} \right)
\]

For the first vapor element
where $k$ is the heat conduction coefficient and $\Delta H_i$ is the partial molar evaporation heat for component $i$. The solubility of air or nitrogen in the droplet is neglected. Equations 14–16 assume thermodynamic equilibrium at the liquid–vapor interface, but diffusive transport on both sides of that interface is accounted for.

Adjustment of the control volume following the updated temperature and concentration determination is achieved by self-tuning based on mass balance and pressure conservation. The volume shrinkage/expansion coefficient, $\eta^V_j$, for liquid layer $j$ over time $\delta t$ can be obtained by

$$
\eta^V_j = \frac{V'_j}{V_j} = \frac{4}{3} \pi (R^{'3}_j - R^{-3}_{j-1}) = \frac{\sum mw_iC'_i}{\sum mw_iC_i} \frac{\rho}{\rho^{'}} \tag{18}
$$

Similarly, the volume shrinkage/expansion coefficient, $\eta^V_j$, for vapor layer $j$ over time $\delta t$ is given by

$$
\eta^V_j = \frac{V'_j}{V_j} = \frac{\sum mw_iC'_i}{\sum mw_iC_i} \frac{T'}{T} \tag{19}
$$

where $\rho$ and $C$ are functions of the composition and temperature and $mw_i$ is the molecular weight for component $i$. Primed and unprimed quantities refer to the properties after or before the time increment, respectively. The volume updating starts first from the element at the droplet center and gradually extends to the outer liquid layer and then to vapor layers while keeping a fixed total gas pressure. The initial conditions are used to update $C$ and $T$ and, therefore, all other transient thermodynamic and transport properties by mass balance (eq 2) and thermal balance (eq 3) for each layer over a time period $t = 0 \rightarrow \delta t$. The boundary conditions are coupled to the updating of the properties for the liquid layer at the droplet center and at the outermost vapor layer throughout the complete simulation. Step by step, all thermodynamic and transport properties for each element can be calculated over the entire evaporation time. The moving-boundary problem is solved explicitly as integration of a series of consecutive fixed-boundary heat transport only, mass transport only, and control volume shrinkage/expansion only steps. The heat transport only and mass transport only problems are solved in the same manner as that for conventional fixed-boundary problems. Only if the grid sizes and time steps are small enough, the treatment should accurately simulate the actual evaporation process.

To save computation, the initial grids are designed with variable size so that the grids are dense near the droplet interface where the gradients are steep. During evaporation, the outer liquid layer (NL) shrinks and finally disappears, while the first vapor layer keeps growing bigger. The decreasing thickness for the liquid layer (NL) requires a much reduced time step, which is proportional to the square of the thickness to avoid oscillations and divergences, resulting in long computing times. The NLth liquid layer needs to be merged with the neighboring (NL – 1)th layer to save computational time. Correspondingly, the growing vapor layer (NL + 1) needs to be divided to enable quick and stable simulation and preserve the accuracy of the finite-difference method. Therefore, a decreasing NL and increasing NV are expected for the moving-boundary problem. The time step can be fixed at a low value or gradually increased with developing temperature and concentration profiles to reduce computational time while ensuring convergence of the simulation.

Valid correlations of the vapor–liquid equilibrium at the interface and accurate correlations for all of the thermodynamic and transport properties are required for reliable simulation. The physical properties for ethanol, water, nitrogen, and air, such as density, heat capacity, viscosity, surface tension, vapor pressure, diffusion coefficient, thermal conductivity, and enthalpy of evaporation, are correlated by polynomial regression of literature data and detailed before. The extended universal quasichemical (UNIQUAC) model accurately predicts and correlates well experimental phase equilibrium diagrams for the saturated or unsaturated ethanol–water–NaCl system. When the NaCl concentration is low, the surface tension, thermal conductivity, viscosity, and evaporation heat for the ethanol–water–NaCl liquid phase are correlated approximately the same as those for the ethanol–water solution described before. The NaCl solubility, heat capacity, density, and component diffusion coefficients for the ethanol–water–NaCl solution are described in the Appendix.

**Results and Discussion**

The code has been validated with evaporation of the water droplet and ethanol droplet. Various parameters have been checked to ensure a fast, convergent, and accurate simulation.

**Water Droplet Evaporation.** The droplet is divided uniformly into a sufficiently large number of grids of a size of $\delta R_L$ to simulate evaporation of a 1048.8-µm-diameter water droplet in still air at atmospheric pressure. A value of 100 for NL is good for fast simulation and accurate description of the droplet temperature inhomogeneity.

**a. Vapor-Phase Grid Distribution.** The code was used to simulate water droplet evaporation in air at atmospheric pressure over 10 s to check the vapor-phase grid distribution effect. For simulation of droplet evaporation in infinite air, to save computation time, the vapor-phase grid size, $R^V_j$ for the jth vapor layer, is designed in the power relationship given below:

$$
R^V_j = R^L + (i - NL)^{1+1/(NL-1)\delta R^V} \delta R^V \tag{20}
$$

where $R^L$ is the droplet size and $\delta R^V$ is the initial first vapor layer thickness. The adjustable factor, $Q$, is selected to reduce the NV and save computation time without compromising the accuracy much. The vapor-phase size is fixed, and $R^V_j/R^L_0$ is set as high as 1132 to simulate the 10 s evaporation in an infinite vapor phase. Four different $Q$ values have been checked, 2, 1, 0.5, and 0.1, corresponding to 28, 150, 800, and 2237 vapor layers. Fewer grids make the computation faster at the price of accuracy. Too many grids require longer computation time. As shown in Figure 2, a value of 800 for NV ($Q = 0.5$) enables convergence and accuracy for the droplet size and surface temperature.

**b. Vapor Grid Size Effect.** To check the vapor grid size effect, five different $\delta R^V_0/\delta R^L_0$ ratios, i.e., 0.5, 1, 2, 5, and 10, have been considered (NV = 800). All of the grid sizes have to be small enough to reduce the error for the finite-difference method. However, too small a grid spacing requires small time
steps and long computation time. As shown in Figure 3, selection of a big grid size will underestimate the temperature depression at the droplet surface. A ratio of 0.5 is sufficient and allows convergence.

c. Vapor Domain Size Effect. The tiny concentration gradient far away from the droplet interface makes it possible to truncate the radius at a certain value without affecting the accuracy for evaporation in infinite still air. $\delta R_v/\delta R_l$ is fixed at 0.5, $Q$ is set as 0.5, and five different values for NV, i.e., 50, 100, 200, 400, and 800, are compared. It is found that for 10 s evaporation a value of 100 for NV is sufficient for convergence. A large value of NV is needed for longer evaporation times to maintain the accuracy.

d. Time Step Effect. A small time step has to be selected based on the grid sizes for convergence and accuracy. However, the smaller the time step, the longer the computing time. The code has been run to simulate 10 s of evaporation with different time steps, i.e., $1 \times 10^{-6}, 1 \times 10^{-5}, 2 \times 10^{-5}, 4 \times 10^{-5}$, and $1 \times 10^{-4}$ s. It is found that a time step of $2 \times 10^{-5}$ s is close to optimal.

e. Evolution of the Temperature, Droplet Size, and Concentration during Evaporation. Figure 5 shows the temperature and water concentration profiles inside the droplet and surrounding vapor phase after 1, 10 and 100 s of evaporation. The temperatures for the droplet and surrounding vapor decrease with time. After 100 s of evaporation, the temperature
is uniform in the droplet because of high water thermal conductivity compared to mass diffusivity. During evaporation, water diffusion continuously increases the vapor-phase water concentration. After 100 s, the water concentration at the vapor-phase boundary is $10^{25}$ times lower than that in the droplet. The selection of a large limited vapor-phase size for numerical simulation of droplet evaporation in infinite air is thus reasonable.

Water droplet evaporation has been simulated for 700 s. As shown in Figure 6, the droplet size decreases with time and it agrees well with experimental data. Theoretically, once the temperature and concentration fields reach steady state, the square of the droplet diameter decreases linearly with time.29 However, our simulation shows that the gradient of the square of the droplet size with respect to time is not constant. The slope decreases with time because the vapor-phase water concentration profile still develops even though the temperature field becomes stable at later times. As shown in Figure 6, the average droplet temperature decreases quickly for the first 20 s and stabilizes close to 279 K. During later evaporation, the temperature depletion slightly continues because the vapor-phase and stabilizes close to 279 K. During later evaporation, the average droplet temperature decreases quickly for the first 20 s and stabilizes close to 279 K.

The ethanol concentration profile in the vapor phase is far from linear in a double-logarithmic plot, indicating that steady-state or quasi-steady-state treatment is not appropriate. The ethanol concentration at the vapor boundary is as low as $10^{-6}$. So, the selection of a large vapor domain instead of an infinite one incurs insignificant error. Unlike evaporation of a droplet in infinite still air, the ethanol droplets move down the chamber at $\sim 10$ m/s. The Sherwood number defined as $Sh = 2K_cR_p/D$ is $\sim 4.2$ instead of 2 for static evaporation, and the Nusselt number is also increased to 3.9 instead of 2.32 The mass and heat transport by convection should have increased the evaporation rate and reduced the droplet temperature. However, interaction of the droplets in the linear streams reduced evaporation. The interaction parameter, $\eta_{int}$, is defined as the relative transfer rate of the droplet in an array to that of an isolated droplet of identical size and composition under identical environmental conditions. Sangiovanni and Labowsky33 found that the interaction parameter for a given geometry is independent of the physical properties of the droplets. The interaction parameters for both heat and mass transport are almost the same, i.e., $\eta_{int} = \eta_{int} = \eta_{int}$. Devarakonda and Ray31,32 obtained a quadratic relationship as follows:

$$\eta_{int} = 0.0611 + 0.0505 \frac{l}{R_p} + 0.0029 \left( \frac{l}{R_p} \right)^2 \quad \text{for } 3 \leq \frac{l}{R_p} \leq 6 \quad (22)$$

where $l$ is the separation distance between neighboring droplets and $R_p$ is the droplet radius. For evaporation, the experimental dimensionless separation distance $ll/R_p$ was 3.5 and the interaction parameter had been determined as 0.27. The change of the droplet size with evaporation time $t$ can be described by the relationship32

$$\frac{dR_p^2}{dt} = \frac{Shn_m \text{MW} \phi}{RTs_p}$$

(23)
were carried by nitrogen gas through a room-temperature drying zone and then a three-zone tubular furnace kept at 400 °C.

**Evaporation in the Drying Zone.** For droplets evaporating in ~40 cm long drying zone, the concentrations for ethanol and water and the temperatures in the vapor phase and droplets change as a result of the heat and mass transport between the droplets and surrounding air. The actual evaporation condition falls between two extremes: isothermal and adiabatic evaporation.

**Isothermal Evaporation.** Considering fast heat transport between the aerosol droplets with room-temperature glass tubing in the drying zone, the temperature at the vapor-phase boundary can be assumed constant at 298.15 K. For an initial 25 °C, 1 µm aerosol droplet with a NaCl mass percentage of 0.01, and a droplet number density of $8 \times 10^7$/cm$^3$, i.e., the radius ratio $R_v/R_f$ ratio is ~15 and the initial ethanol concentration is set as 70% (v/v), the evaporation is very fast. As shown in Figure 9, the droplet surface temperature decreases quickly at the beginning to 292.6 K and then rebounds to 298.15 K. After ~0.01 s of evaporation, the vapor phase is at equilibrium with the liquid phase. No further change with the droplet size, temperature, and concentration occurs. The concentration, NaCl saturation ratio, and temperature are uniform within the droplet or in the vapor phase. The final saturation ratio is 0.178.

**Adiabatic Evaporation.** As a comparison, adiabatic evaporation of 1-µm-radius aerosol droplets in a nitrogen layer of 30 µm radius; i.e., a droplet concentration of $10^7$/cm$^3$ has been simulated for aerosol droplets with an initial system temperature of 25 °C and an ethanol concentration of 70% (v/v). The initial NaCl mass fraction is set at 0.02. Figure 10 shows evolution of the droplet size and surface temperature with the evaporation time separately. After ~0.028 s of evaporation, the system reaches equilibrium. The concentrations are homogeneous in both the liquid droplet and the vapor phase. The temperature is uniform, 275 K in both liquid and vapor phases. After adiabatic evaporation, the NaCl mass fraction is ~0.077 45 in the droplets.

For evaporation of the droplets of 2 µm radius, if the initial concentration, temperature, $R_v/R_f$ ratio, and initial NaCl mass fraction are kept the same as those for the adiabatic evaporation of 1 µm droplets, the equilibrium time is increased from ~28 to ~100 ms. The final temperature, droplet size, and NaCl mass fraction are exactly the same as those for a 1 µm droplet.

**Ethanol Concentration Effect.** To check the effect of the ethanol concentration on evaporation, the initial ethanol concentration is increased to 80% (v/v) for the droplets of a radius of 2 µm with the same $R_v/R_f$ ratio, temperature, and NaCl mass fraction as that for evaporation with 70% (v/v) droplets. The final temperature is 274.2 K and the equilibration time ~0.11 s, as shown in Figure 11. Compared to evaporation of 2-µm-radius, 70% (v/v) droplets, increasing the initial ethanol concentration increases the reduction in the droplet temperature and droplet size. The final NaCl mass percentage at equilibrium increases from 0.077 45 to 0.108 as a result of increased solvent evaporation.

Evaporation in the drying zone is a very fast process. A relatively long residence time of ~2 s allows sufficient time for heat transfer between the droplets and the carrier gas. We conclude that at the time that the droplets enter the heating zone the temperature can be assumed to be at room temperature and the vapor phase is at equilibrium with the liquid phase. The temperature and concentration are uniform in both the vapor and liquid phases; i.e., the vapor phase and droplets can be treated as at equilibrium after single-stage flash evaporation at 25 °C.
Evaporation in the Heating Zone. In the heating zone, the temperature increases gradually from room temperature to 400 °C. The evaporation process can be assumed to consist of consecutive evaporation at increasing temperature over varying residence time. To illustrate the effect of evaporation on nucleation and crystallization, we simplify the problem and select 100 °C as the heating temperature. Evaporation in the heating zone is simulated for ethanol-water-NaCl droplets of an initial temperature of 25 °C and a droplet radius of 1 µm. The radius ratio of the vapor phase to the liquid phase \(R_V/R_L\) is set at 30, and the NaCl mass fraction is set at 0.08. The ethanol concentration is set at 60% (v/v). The vapor phase is in equilibrium with the droplet; i.e., the partial pressure is 0.0483 atm for ethanol and 0.0267 atm for water. The total pressure is 1 atm, with N\(_2\) as the balance. It is assumed that NaCl nucleation and crystallization follow a homogeneous nucleation mechanism. The first nucleation and crystallization happen at the location where the saturation ratio exceeds a critical saturation ratio (a representative value of 1.3 was selected). Following \(\sim 0.183\) ms of evaporation, the NaCl concentration becomes supersaturated with a maximum saturation ratio of 1.3 at the center. The temperature distribution is shown in Figure 12. The mole fraction distributions for ethanol, water, NaCl, and N\(_2\) are shown in Figure 13. With evaporation, the concentrations for water and NaCl are enriched at the surface. The temperature profile in the liquid phase is uniform because of the large value of the Lewis number \(Le\), which is a dimensionless number defined as the ratio of thermal diffusivity to mass diffusivity. After 0.183 ms of evaporation, crystallization starts at the droplet center, and NaCl is assumed to be saturated at the center. The heat of crystallization is neglected because the heat of evaporation is much larger than the heat of crystallization and the thermal diffusivity is much higher than the mass diffusivity in the liquid phase. Quick thermal conduction makes the temperature quite uniform within the droplets during crystallization. As shown in Figure 14, even though solvent evaporation induces enrichment of NaCl at the surface, relatively quick diffusion of NaCl compared to solvent molecules and increased NaCl solubility due to depletion of the antisolvent (ethanol) for NaCl near the droplet surface enable the NaCl saturation ratio to be maximum at the droplet center. During evaporation, the droplet surface temperature increases quickly from the initial.

Figure 9. Droplet surface temperature change versus isothermal evaporation time (left). Droplet size change versus isothermal evaporation time (right).

Figure 10. Droplet size change versus adiabatic evaporation time (left). Droplet surface temperature change versus adiabatic evaporation time (right).
298.15 K to near 327 K, as shown in Figure 12. Figure 15 demonstrates evolution of the NaCl saturation ratio profile at the beginning of stable nucleus formation and after 28.7, 61.6, 96.3, and 131.6 µs of crystallization. Once nucleation and crystallization occur at the center, because the crystallization process is very quick and controlled by diffusion, the NaCl concentration is saturated at the crystal surface. This always keeps the saturation ratio at unity at the center and results in a sharp NaCl concentration gradient near the droplet center. Diffusion of NaCl to the center reduces the nearby NaCl concentration and decreases the saturation ratio. As evaporation proceeds, the droplet size decreases and the surface NaCl concentration keeps increasing after 28.7, 61.6, and 96.3 µs of crystallization. After 131.6 µs of crystallization, the saturation ratio reaches a maximum at the surface. The value is below the critical saturation ratio and continues to decrease with further evaporation, making it impossible to form additional stable nuclei in the droplet. The fast diffusion depletes NaCl in the droplet and quenches the formation of new nuclei, favoring one single crystal at the center. It takes only ∼0.3 ms for the droplets from entering the heating zone to forming only a stable single crystal at the droplet center, corresponding to ∼0.026 mm axial movement of aerosol droplets in the reactor tube. It is reasonable to select 100 °C as the representative heating temperature because the choice of a lower temperature, or treatment as a sequence of consecutive evaporation steps with increasing heating temperature, does not affect the conclusion. According to the simulation, if the droplet size and/or evaporation rate are sufficiently large, NaCl cannot diffuse quickly enough to the central nucleus to avoid exceeding the critical saturation in the droplet bulk, indicating that multiple NaCl nanocrystals will be formed in the final particles.

It should be pointed out that the initial droplet size, precursor composition, evaporation conditions, solubility, diffusion, vapor pressure, and heat transport all couple together in controlling the final particle morphology and structure. A combination of good solvent and antisolvent will favor the formation of only a single salt crystal or core inside each particle. Under aerosol generation and synthesis conditions, the droplet size is small and the growing assembled surfactant/silica layer provides a
heat/mass-transfer barrier. The reduced evaporation rate and droplet surface temperature decrease the NaCl gradient. In addition, continuous formation of ethanol as a product of tetraethylorthosilicate hydrolysis sustains the ethanol concentration inside the droplet and maintains the ethanol gradient critical to the initial NaCl nucleation/crystallization at the center.

For a comparison, an ethanol-free precursor composed of 4.0 g of cetyltrimethylammonium bromide, 10.42 g of tetraethoxysilane (TEOS), 78.7 g of deionized water, and 1.10 g of 1 N HCl was used for aerosol-assisted EISA under the same conditions as those for Figure 8. Without the addition of ethanol, sol–gel reactions occur rapidly and less ordered silica particles with multiple NaCl crystals are formed (Figure 16). The delayed NaCl nucleation and crystallization and higher NaCl concentration favor the formation of multiple smaller NaCl crystals instead of a bigger single one.

Engineering of the Aerosol Particle Architecture. Inspired by the simulation, we fabricated aerosol particles of varying architectures. It is expected that high solvent evaporation temperatures lead to less dense particles and hollow particles, whereas low evaporation temperatures and initial concentrations near saturation favor the formation of dense solid particles. Figure 17 shows hollow cerium oxide/chloride particles fabricated by the aerosol synthesis of CeCl₃–ethanol droplets at 400 °C. We sprayed a metal oxide precursor in a hydrophobic low-boiling-point solvent (hexane) at room temperature. Quick evaporation enriches the precursor concentration at the droplet surface, where hydrolysis and condensation of CeCl₃ results from the reaction with water supplied from the vapor phase. Because water diffusion is limited by the solubility of the hydrophobic solvent, this “sol gel” reaction occurs only at the droplet surface. In this way, we demonstrated a general facile
room-temperature fabrication method for hollow spherical metal oxide particles. The low-temperature technique works for encapsulation of heat-sensitive drug materials. It is possible to fabricate hollow spherical particles at a low temperature even using a hydrophilic solvent if we increase the evaporation rate using a low-boiling solvent or operate under vacuum to suppress the droplet surface temperature and induce nucleation/crystalization to occur first at the droplet surface instead of the center.

It is found by the simulation that a lower salt solubility requires slower evaporation and/or a smaller droplet size to homogenize the salt concentration or else multiple stable nuclei and multicrystals will be formed. We have made single and multiple cerium compound cores in silica particles by selecting the solvents with varying solubility (Figure 18). In addition to the controls by solvent hydrophobicity, solubility, and evaporation rate, according to the simulation, if the salt ions have a high affinity for the gelling silica matrix, the much reduced diffusion coefficient of the ions will promote the formation of well-dispersed small uniform crystals in the matrix. Applying these principles, we also invented a technique for antimicrobial agents in which silver nanoclusters are uniformly dispersed in silica particles (Figure 19).

Concluding Remarks

A simulation methodology/computer code has been developed for the study of the evaporation of multicomponent droplets. The code successfully describes water droplet and ethanol droplet evaporation and clearly demonstrates the accompanying evolution of composition, temperature, droplet size, and inhomogeneities. For ethanol–water–NaCl droplet evaporation, the aerosol mist can be assumed to be at equilibrium before entering the heating zone. After entering the heating zone, NaCl nucleation and crystallization occur quickly, starting first at the center. Crystallization depletes the surrounding region of NaCl, and only a single crystal is formed within each particle. The
code can be used to predict profiles for the temperature, size, and compositions for both pure and multicomponent evaporating droplets. So far, the temperature and concentration profiles within the droplets are difficult to study experimentally. The computer code provides sufficient accuracy for engineering design and provides a guideline for aerosol research such as evaporation-induced self-assembly and fabrication of nanostructured aerosol particles.

Appendix

Heat Capacity. The heat capacity for an ethanol (1)–water (2)–NaCl (3) solution is given as

\[ C_p = C_p^0(X_1 + X_3) + X_1C_{p1} + X_2C_{p2} + X_3C_{p3} \]  
(A-1)

where the excess heat capacity \( C_{p}^{e} \) was given by Benson and Darcy.\(^{36}\)

Density. The density \( \rho (\text{kg/m}^3) \) for an ethanol–water–NaCl solution is given as\(^{37}\)

\[ \rho = 1000(A_0 + A_1w_1 + A_2w_2 + A_3w_3 + A_4w_1w_3^2) \times \exp \left[ \frac{A_5w_1}{w_3} + \frac{A_6w_2^2}{w_3^2} \right] \]  
(A-2)

NaCl Solubility. In the temperature range 298–323 K, the solubility, \( S_{NaCl} \) (mol/kg), is given by Farelo et al.\(^{38,40}\)

\[ S_{NaCl} = \frac{0.0956424 \exp(0.00725127(1 - X))}{0.0956424 \exp(0.00725127) + X} \left( 7.78461 - 0.01566T + 0.000034T^2 \right) \]  
(A-3)

where \( X \) is the ethanol mass fraction in a salt-free solvent.

Diffusion Coefficients. The Vignes relationship\(^{41}\) is used to determine binary diffusion coefficients at finite concentrations. The diffusion coefficients for ethanol and water can be assumed to be the same as those for a binary system, and the infinite diffusion coefficient for NaCl, \( D_{12}^{0} \), can be estimated by the Wilke–Chang estimation\(^{42,43}\)

\[ D_{12}^{0} = \frac{7.4 \times 10^{-8}(\phi M_2)^{1/2}T}{\eta_1 V_1^{0.6}} \]  
(A-4)

For an ethanol–water solution, we replace the molecular weight term in eq A-4 with

\[ \phi M = \sum x_i \phi_i M_i \]  
(A-5)

For NaCl concentrations up to 2 M, the diffusion coefficient can be estimated by an empirical equation proposed by Gordon.\(^{44}\)

\[ D_{12} = D_{12}^{0} \eta_1(\rho S)\left( 1 + m \frac{\partial \ln y_1}{\partial m} \right) = D_{12}^{0} \left( 1 + m \frac{\partial \ln y_1}{\partial m} \right) \]  
(A-6)

where \( m \) is the molarity of the solute (mol/kg of solvent), \( y_1 \) the mean activity coefficient of the salt, \( \rho \) the molar density of the solvent, \( V_i \) the partial molar volume of the solvent, \( \eta_1 \) the viscosity of the solvent, and \( y_1 \) the viscosity of the solution.

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